where

$$E_{+} = e^{-\Gamma_{mn}(b_1 + a_1)}$$

$$E_{-} = e^{-\Gamma_{mn}(b_1 - a_1)}$$

and similar expressions are found for F_{11} and F_{22}. It is clear that these expressions are rapidly evaluated without recourse to Bessel or Hankel function computation.

III. Accuracy of Approximation

Calculations of impedance were carried out for a junction between a coaxial line and rectangular waveguide, using the square contour approximation, and the results compared with previously published experimental measurements [3]. The infinite series was truncated at $m = 3A/2a$ and $n \leqslant 10$.

Fig. 2 shows the comparison for $Z_c = 50\ \Omega$, using (2) and (5), both of which give good accuracy. For $Z_c < 50\ \Omega$, Fig. 3(a) shows (5) gives good accuracy, but (2) shows some error; when $Z_c > 50\ \Omega$, Fig. 3(b) shows that both (2) and (5) give good accuracy.

Overall, the comparison with experimental measurement shows that (5) yields good accuracy, with the square-contour approximation, and that it provides a useful alternative to use of the circular contours for the coaxial-line cross-sectional boundaries.

IV. Conclusion

Although the approach developed here has been applied only to rectangular waveguide junctions, it is evident from the formulation that it will have useful application in a wide range of waveguides having conducting planes on their upper and lower surfaces. The key element in the analysis is derivation of a dyadic Green's function for the waveguide, accomplished readily if a complete set of modes can be found for that guiding structure; note that only one component of the dyadic Green's function is required in this analysis.

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Coupled Microstrips on Double Anisotropic Layers

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***Abstract*—A variational expression is presented for the mode capacitances of coupled microstrip lines with double anisotropic layered substrates. It is shown that the even- and odd-mode phase velocities can be made equal when a sapphire layer is deposited on a boron nitride substrate. High coupler directivity can be achieved with appropriate values of both layers' thicknesses.**

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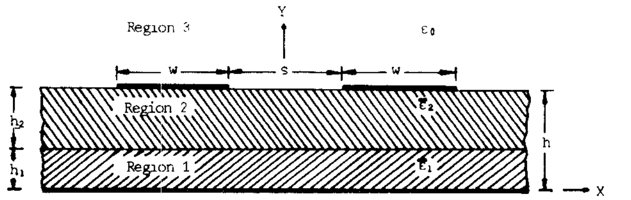

Fig. 1 Cross section of coupled microstrips on double anisotropic substrate layers.

I. Introduction

The analysis of structures such as single [1]–[4] and coupled [5]–[11] microstrip lines, slot lines [12], and coplanar waveguides [13] on anisotropic substrates, has received considerable attention in the last few years as a consequence of the interesting properties of crystalline substrate materials, such as sapphire [1] and boron nitride, or some glass- and ceramic-filled polymeric materials, e.g. Duroid and Epsilam, which are electrically anisotropic substances [10].

Certain types of anisotropy can be advantageous for coupled microstrip components. Alexopoulos and Krowne show in [7] that the difference between the odd- and even-phase velocities will be reduced if the permittivity in the direction parallel to the ground plane is greater than the perpendicular tensor component, in an anisotropic substrate. To equalize the phase velocities, the parallel component must be approximately twice the perpendicular component. This anisotropy is nonexistent in practical microwave materials [10]. Kobayashi and Terakado [8] propose to equalize the phase velocities by rotating the crystal axes an angle θ with respect to the microstrip axes. Recently, Alexopoulos and Maas [10], [14] have demonstrated that high coupler directivity can be achieved by making use of the substrate anisotropy in conjunction with a top cover to equalize even- and odd-mode phase velocities.

In this paper, coupled microstrip lines in which the substrate is made with two layers of anisotropic dielectrics are studied. An accurate expression to compute the quasi-static characteristic parameters of this structure has been obtained.

A layer of sapphire on another of boron nitride has been used as a composite substrate to equalize the even- and odd-mode phase velocities. Therefore, the coupler directivity can be improved by making use of this substrate with appropriate values of both layers' thicknesses.

II. Analysis Method

Fig. 1 shows the coupled microstrip lines to be analyzed. The substrate is composed of two layers of anisotropic dielectric (region 1 and 2).

The anisotropy is described by two permittivity tensors $\bar{\epsilon}_1$ and $\bar{\epsilon}_2$, respectively, which are given by

$$\bar{\epsilon}_i = \epsilon_0 \begin{bmatrix} \epsilon_i^{11} & \epsilon_i^{12} \\ \epsilon_i^{12} & \epsilon_i^{22} \end{bmatrix} \qquad i = 1,2. \tag{1}$$

It is assumed that the strips are infinitely thin and that the conductors are perfect.

In order to compute the characteristic parameters of this structure, we have extended, in this paper, the quasi-static approach which was used in [4], [9], [11] to obtain a variational expression for the modal capacitances.

To this end, the solution of the Laplace's equation

$$\vec{\nabla} \cdot (\bar{\epsilon}_i \cdot \vec{\nabla} \psi_i) = 0 \tag{2}$$

in the spectral domain for the anisotropic region i can be written as

$$\tilde{\psi}_i(\beta, y) = (A_i \sinh(\beta R_i y) + B_i \cosh(\beta R_i y)) \exp(-j\beta S_i y) \quad (3)$$

where $\tilde{\psi}_i(\beta, y)$ is the Fourier transform of the potential function $\psi_i(x, y)$ and A_i and B_i are constants which would be determined by applying the boundary conditions. R_i and S_i are given by

$$R_i = \left(\epsilon_i^{11}/\epsilon_i^{22} - (\epsilon_i^{12}/\epsilon_i^{22})^2\right)^{1/2} \quad (4)$$

$$S_i = \epsilon_i^{12}/\epsilon_i^{22}. \quad (5)$$

In the present analysis, by applying the boundary conditions, a relation between the Fourier transforms at the interface 2—3 potential is obtained as

$$\tilde{V}_{e,o}(\beta) = 2\int_0^{\infty} V_{e,o}(x) \begin{Bmatrix} \cos(\beta x) \\ \sin(\beta x) \end{Bmatrix} dx \quad \begin{matrix} \text{even mode} \\ \text{odd mode} \end{matrix}. \quad (6)$$

The transforms of the surface charge density on the strips are given by

$$\tilde{\rho}_{e,o}(\beta) = 2\int_{s/2}^{s/2+w} \rho_{e,o}(x) \begin{Bmatrix} \cos(\beta x) \\ \sin(\beta x) \end{Bmatrix} dx \quad \begin{matrix} \text{even mode} \\ \text{odd mode} \end{matrix} \quad (7)$$

and therefore

$$\tilde{V}_{e,o}(\beta) = g(\beta)\tilde{\rho}_{e,o}(\beta)/\epsilon_0 \quad (8)$$

where

$$g(\beta) = \left\{\beta\left(1 + \epsilon_2^{eq} \frac{\epsilon_2^{eq} + \epsilon_1^{eq}\coth(\beta h_1^{eq})\coth(\beta h_2^{eq})}{\epsilon_2^{eq}\coth(\beta h_2^{eq}) + \epsilon_1^{eq}\coth(\beta h_1^{eq})}\right)\right\}^{-1} \quad (9)$$

with

$$\epsilon_i^{eq} = \left(\epsilon_i^{11}\epsilon_i^{22} - (\epsilon_i^{12})^2\right)^{1/2} \quad (10)$$

$$h_i^{eq} = R_i h_i, \qquad i = 1, 2. \quad (11)$$

The above results are useful for obtaining expressions for the mode capacitances C_e and C_o.

By applying Perseval's theorem and (8), the electric energy per unit length of the structure can be written as

$$U_{e,o} = (2\pi\epsilon_0)^{-1}\int_0^{\infty} |\tilde{\rho}_{e,o}|^2 g(\beta)\, d\beta \quad (12)$$

and the mode capacitance

$$C_{e,o}^{-1} = (2\pi Q^2\epsilon_0)^{-1}\int_0^{\infty} |\tilde{\rho}_{e,o}|^2 g(\beta)\, d\beta \quad (13)$$

where Q is the charge per unit length on the strip.

Equation (13) is an analytic expression for the lower bound of the mode capacitances of coupled microstrip lines on a two-layer anisotropic dielectric and is identical to that obtained in [9] when h_1 or $h_2 = 0$, or $\bar{\epsilon}_1 = \bar{\epsilon}_2$.

Expression (13) is also in agreement with previous results [5]. If region 1 is air ($\epsilon_1^{eq} = 1$, $h_1^{eq} = h_1 \to \infty$) (9) is reduced to (13) in [5] when $\epsilon_2^{12} = 0$.

A) Variational Approach

In order to compute (13), we first expand the unknown $\rho_{e,o}(x)$ in terms of known basis functions $\phi_i(x)$, i.e.,

$$\rho_{e,o}(x) = \sum_0^N a_i \phi_i(x). \quad (14)$$

Next, we substitute (14) into (7) and then into (13). Then, the Rayleigh-Ritz procedure is used. The following system of equations is obtained:

$$\sum_1^N a_i \cdot \xi_{ij} = -\xi_{0j} \quad (15)$$

where

$$\xi_{ij} = \int_0^{\infty} \xi_i(\beta)\xi_j(\beta) g(\beta)\, d\beta, \qquad i, j = 0, 1, \cdots, N \quad (16)$$

with

$$\xi_0 = \int_{s/2}^{s/2+w} \{\phi_0(x)/\lambda_0\} \begin{Bmatrix} \cos(\beta x) \\ \sin(\beta x) \end{Bmatrix} dx \quad \begin{matrix} \text{even mode} \\ \text{odd mode} \end{matrix} \quad (17)$$

$$\xi_i = \int_{s/2}^{s/2+w} \left\{\phi_i(x) - \frac{\lambda_i}{\lambda_0}\phi_0(x)\right\} \begin{Bmatrix} \cos(\beta x) \\ \sin(\beta x) \end{Bmatrix} dx \quad \begin{matrix} \text{even mode} \\ \text{odd mode} \end{matrix} \quad (18)$$

and

$$\lambda_i = \int_{s/2}^{s/2+w} \phi_i(x)\, dx, \qquad i = 0, 1, \cdots, N. \quad (19)$$

Solving (15), the mode capacitances (lower bounds) can be expressed as (16) in [9]

$$C_{e,o} = (\epsilon_0\pi/2)\left(\xi_{00} + \sum_1^N a_i \xi_{0i}\right)^{-1}. \quad (20)$$

B) Basis Functions

The basis functions $\phi_i(x)$ in the expansion (14) which have been assumed in this paper are

$$\phi_0(x) = 1/w \qquad d \leqslant x \leqslant c \quad (21a)$$

$$\phi_1(x) = (1/w)\left\{-\ln\left(\frac{x-d}{w}\right)\right\} \qquad d \leqslant x \leqslant c \quad (21b)$$

$$\phi_2(x) = (1/w)\left\{-\ln\left(\frac{c-x}{w}\right)\right\} \qquad d \leqslant x \leqslant c \quad (21c)$$

$$\phi_{n>2}(x) = (1/w)\left\{\frac{x-t}{w/2}\right\}^{n-2} \qquad d \leqslant x \leqslant c \quad (21d)$$

where

$$d = \frac{s}{2}, \quad c = \frac{s}{2} + w, \quad t = \frac{s}{2} + \frac{w}{2}.$$

This trial function $\rho_{e,o}(x)$ satisfies the appropriate edge conditions and the computation of $C_{e,o}$ for these basis functions has good accuracy.

In order to estimate the error in this computation, we have compared, in Table I, the results given in [8] for coupled microstrips on anisotropic substrate with those obtained from (20) ($\bar{\epsilon}_1 = \bar{\epsilon}_2$) using (21), for $N = 2$, 4, 6, and 8. The difference between both results is very small, about 0.2 percent for $N = 8$ in the odd mode.

III. Results Application in Coupler Design

Equation (20) has been used in this paper to compute the characteristics of coupled microstrip lines with a substrate made of two layers, one of boron nitride (region 1), with $\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3.40$, $\epsilon_1^{12} = 0$, and another of sapphire (region 2), $\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11.6$, $\epsilon_2^{12} = 0$. Fig. 2 shows the variation in the even- and odd-mode normalized phase velocities v_e/c, v_o/c, as the relation between the thickness of both anisotropic layers, h_2/h is changed.

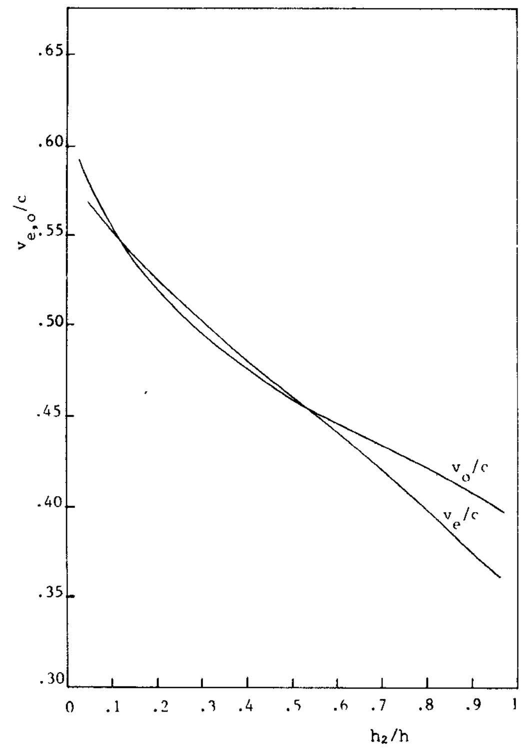

Fig. 2. Even- and odd-mode normalized phase velocities v_e/c, v_o/c versus h_2/h for coupled microstrips on double anisotropic layers, $w/h = 1.0$, $s/h = 1.0$. Region 1, boron nitride ($\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3.40$, $\epsilon_1^{12} = 0$). Region 2, sapphire ($\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11.60$, $\epsilon_2^{12} = 0$).

TABLE I
EVEN- AND ODD-MODE CAPACITANCES OF COUPLED MICROSTRIPS ON SAPPHIRE

θ		$C_{e,o}/\epsilon_0$ by present method				$C_{e,o}/\epsilon_0$ by
		N=2	N=4	N=6	N=8	Kobayashi
0°	even mode	15.67	16.32	16.36	16.37	16.37
	odd mode	24.39	25.82	25.97	25.98	26.04
30°	even mode	16.28	16.96	17.01	17.02	17.02
	odd mode	24.67	26.14	26.29	26.30	26.36

θ, angle between crystal and microstrip axes.
$w/H=1$, $s/H=.4$, $\epsilon_1^{11}=\epsilon_2^{11}=11.6$, $\epsilon_1^{22}=\epsilon_2^{22}=9.40$
$\epsilon_1^{12}=\epsilon_2^{12}=0$

It can be observed that there are two values of h_2/h (0.15 and 0.52 in this case) that equalize the phase velocities. Fig. 3 shows the values of h_2/h when $v_e = v_o$ for s/h equals 0.1, 0.2, 0.3, 0.4, and 0.5 versus w/h.

The corresponding (only for the upper values of h_2/h) even- and odd-mode impedances Z_e, Z_o are presented in Fig. 4.

These results show that the configuration of two microstrips on double anisotropic layers can be useful in directional coupler design. For these objects, similar graphics to those shown in Figs. 3 and 4 can be used. For each design s/h and w/h are adjusted until the desired Z_o, Z_e are achieved in the graphic of Fig. 4. The proper value of h_2/h is obtained from Fig. 3. If a more exact adjustment is desired, we can vary s/h and w/h around the previous approximate values until the desired precision is obtained.

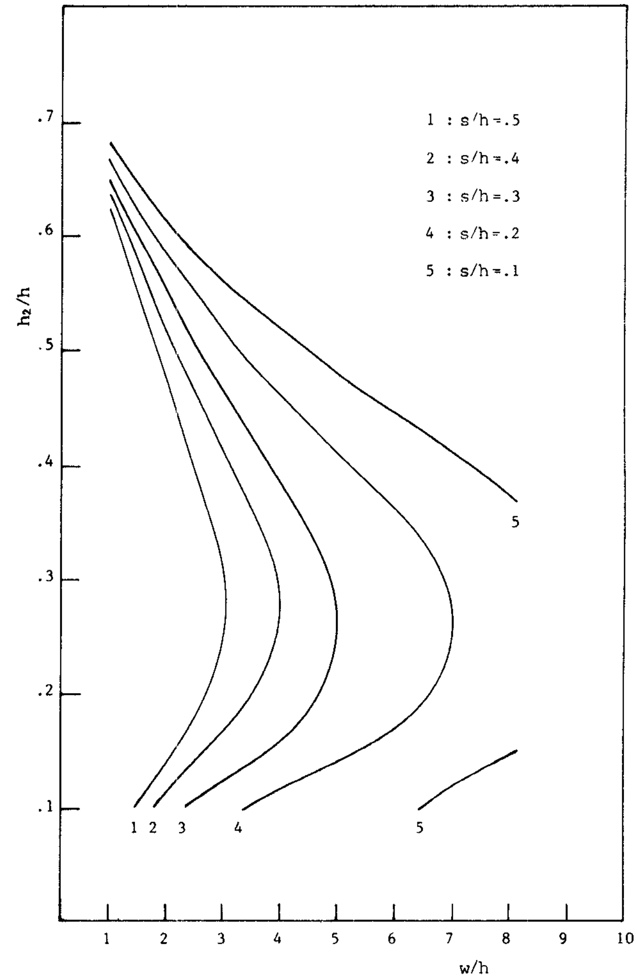

Fig. 3 Layer thickness relation h_2/h for $v_e = v_o$ versus w/h, $s/h = 0.1$, 0.2, 0.3, 0.4, and 0.5. Region 1, boron nitride ($\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3.40$, $\epsilon_1^{12} = 0$). Region 2, sapphire ($\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11.60$, $\epsilon_2^{12} = 0$).

In this paper s/h, w/h, and h_2/h were adjusted for a 10-dB coupler in a 50-Ω system, using the previous procedure. With $s/h = 0.37$ and $w/h = 1.05$, the desired $Z_o = 36.0\Omega$ and $Z_e = 69.3\Omega$ were achieved. For $h_2/h = 0.645$, $v_e/c = 0.4351$, $v_o/c = 0.4352$. The directivity is in this case 66 dB.

Fig. 5 shows the effect of changing the relation h_2/h on $Z_{e,o}$ and $v_{e,o}$. The even-mode impedance is somewhat more sensitive to a change in the relation h_2/h than the odd-mode impedance, and v_o changes more slowly than v_e for high values of h_2/h. Consequently, the equalization of mode phase velocities can be achieved with adequate values of h_2/h. The 10-dB coupler directivity versus h_2/h is shown in Fig. 6.

Finally, it has been determined that, as could be suspected from the equivalence formulas (10) and (11), the equalization of mode phase velocities described in this paper for the coupled microstrips with a two-layer anisotropic substrate can be achieved with a two-layer isotropic substrate or even with a mixed isotropic–anisotropic structure, provided that the equivalent permittivity of the upper layer material was approximately twice or more than the one of the lower layer material.

IV. CONCLUSIONS

A spectral and variational analysis is presented for obtaining an accurate expression useful to compute the modal capacitances (lower bounds) for coupled microstrip lines on substrates made

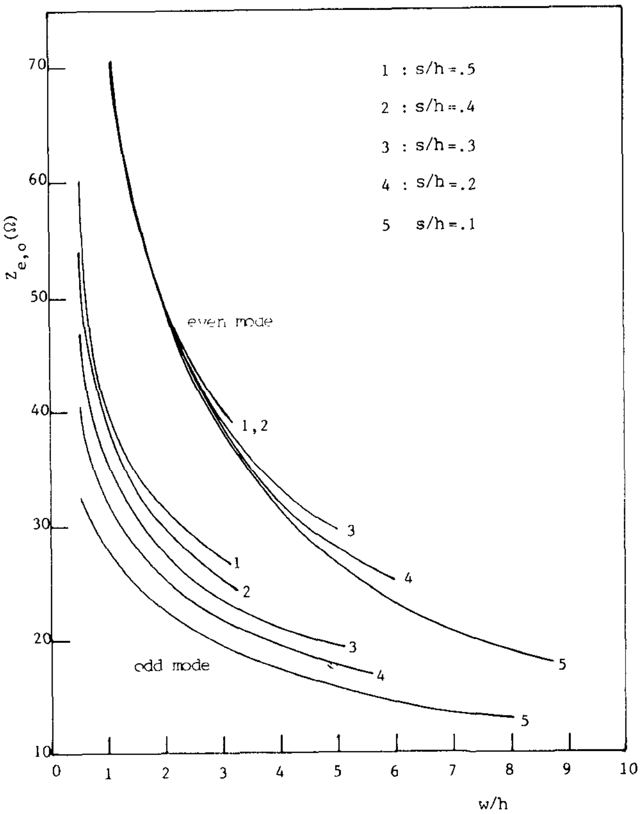

Fig 4. Even- and odd-mode characteristic impedances Z_e, Z_o for $v_e = v_o$ versus w/h, $s/h = 0.1$, 0.2, 0 3, 0.4, and 0.5. Region 1, boron nitride ($\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3\ 40$, $\epsilon_1^{12} = 0$). Region 2, sapphire ($\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11\ 60$, $\epsilon_2^{12} = 0$)

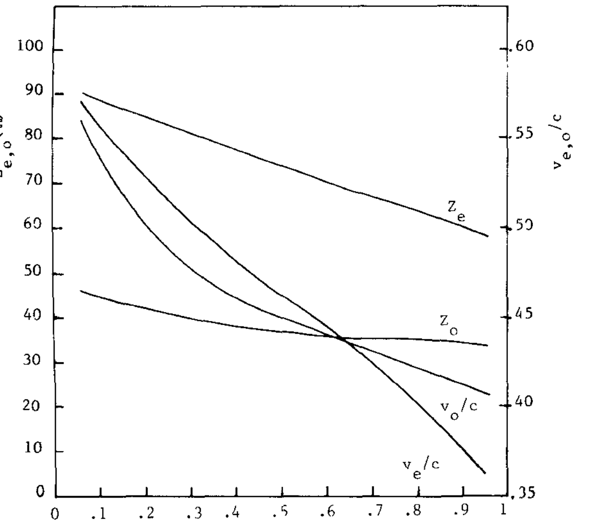

Fig. 5 10-dB coupler sapphire-boron nitride substrate microstrip parameters versus h_2/h, $w/h = 1.05$, $s/h = 0.37$. Region 1, boron nitride ($\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3\ 40$, $\epsilon_1^{12} = 0$) Region 2, sapphire ($\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11.60$, $\epsilon_2^{12} = 0$).

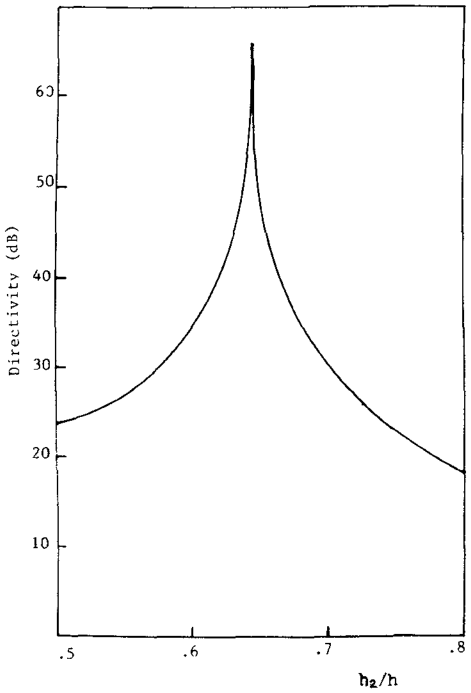

Fig. 6 10-dB coupler sapphire-boron nitride substrate microstrip directivity versus h_2/h. Region 1, boron nitride ($\epsilon_1^{11} = 5.12$, $\epsilon_1^{22} = 3.40$, $\epsilon_1^{12} = 0$). Region 2, sapphire ($\epsilon_2^{11} = 9.40$, $\epsilon_2^{22} = 11.60$, $\epsilon_2^{12} = 0$).

with two layers of anisotropic dielectrics. This expression can be employed to calculate the characteristic parameters of these lines. It has been shown that the even- and odd-mode phase velocities can be equaled if the combination of a sapphire layer on boron nitride is used as substrate. Thus, these coupled microstrip lines can be useful in the design of directional couplers. High directivity can be achieved using appropriate values of both layers' thicknesses.

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